

# Lensing reconstruction from *Planck* sky maps: inhomogeneous noise

Duncan Hanson,<sup>1\*</sup> Graca Rocha<sup>2,3</sup> and Krzysztof Górski<sup>2,4</sup>

<sup>1</sup>*Institute of Astronomy and Kavli Institute for Cosmology Cambridge, University of Cambridge, Madingley Road, Cambridge CB3 0HA*

<sup>2</sup>*Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, CA 91109, USA*

<sup>3</sup>*Department of Physics, California Institute of Technology, Pasadena, CA 91125, USA*

<sup>4</sup>*Warsaw University Observatory, Aleje Ujazdowskie 4, 00478 Warszawa, Poland*

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## ABSTRACT

We discuss the effects of inhomogeneous sky coverage on cosmic microwave background lens reconstruction, focusing on application to the recently launched *Planck* satellite. We discuss the ‘mean field’ which is induced by noise inhomogeneities, as well as three approaches to lens reconstruction in this context: an optimal maximum-likelihood approach which is computationally expensive to evaluate and two suboptimal approaches which are less intensive. The first of these is only suboptimal at the 5 per cent level for *Planck*, and the second prevents biasing due to uncertainties in the noise model.

**Key words:** gravitational lensing – methods: numerical – cosmic microwave background – cosmology: observations.

## 1 INTRODUCTION

The current generation of cosmic microwave background (CMB) data has proven to be remarkably well approximated as a statistically isotropic Gaussian random field (Komatsu et al. 2009). Upcoming experiments are expected to push decisively past this approximation to reveal a CMB which has been subtly distorted by gravitational lensing due to the large-scale structure (LSS) which intercedes between ourselves and the surface of last scattering (Lewis & Challinor 2006). Mathematically, the effect of a fixed LSS realization is to make the CMB statistically anisotropic, introducing off-diagonal elements into its covariance. This enables one to construct estimators for the lensing potential (Hirata & Seljak 2003; Okamoto & Hu 2003). The power spectrum of the measured lensing potential may then be used to obtain improved parameter constraints, particularly for parameters which affect the late-time evolution of the Universe. The recently launched *Planck* satellite, for example, is expected to measure the CMB lensing signal internally with cosmologically useful precision, enabling it to constrain the sum of neutrino masses to 0.1 eV (Lesgourgues et al. 2006). Lensing is also important as a potential contaminant for non-Gaussianity studies. The cross-correlation between the lensing potential and Integrated Sachs-Wolfe (ISW)/Rees–Sciama induced temperature fluctuations results in a bispectrum which has large overlap with the ‘local’ type of non-Gaussianity. Lensing therefore results in a bias to primordial non-Gaussianity estimation, which will be significant for *Planck* (Serra & Cooray 2008). Correction for this bias will be aided by an accurate lensing reconstruction.

CMB lensing reconstruction works on the assumption that the underlying CMB is statistically isotropic, and that any statistical anisotropy is due to gravitational lensing. As such, it is potentially contaminated by any systematic which introduces anisotropy on to the observed sky: beam asymmetries, astrophysical foregrounds and inhomogeneous sky coverage are all expected to complicate the lens reconstruction. In the absence of computationally expensive deconvolution map making (Armitage & Wandelt 2004), beam asymmetries represent an unavoidable source of systematic error, which will need to be quantified for any ultimate lensing analysis with *Planck*. Foregrounds, on the other hand, may be cleaned to a high degree of accuracy by exploiting *Planck*’s wide frequency coverage. The magnitude of residual foregrounds at the small scales of interest to lensing reconstruction will not be adequately understood until *Planck* has started to collect data, however. In this work, we will focus on the effects of inhomogeneous sky coverage. The scan strategy of *Planck* will result in noise levels which depend strongly on ecliptic latitude, and so the effects of noise inhomogeneities are a large concern. We will study both the optimal treatment of noise inhomogeneities, as well as two suboptimal approaches: one which is computationally simpler than the optimal reconstruction and the other which is insensitive to the instrumental noise model.

### 1.1 Lens reconstruction

We begin by reviewing the methodology of lens reconstruction. Consider a data model given by

$$\hat{\Theta}(\Omega) = \Theta[\Omega + \nabla\Phi(\Omega)] + n(\Omega), \quad (1)$$

where  $\hat{\Theta}$  is the observed CMB,  $\Omega$  picks out a location on the unit sphere,  $\Phi$  is the lensing potential (for more details see Lewis &

\*E-mail: dhanson@ast.cam.ac.uk

Challinor 2006),  $\Theta$  is the primary, unlensed CMB with covariance  $C^{\Theta\Theta}$  and  $n(\Omega)$  is the instrumental noise realization with covariance matrix  $C^{nn}$ . Throughout this work, we will use a fixed flat  $\Lambda$  cold dark matter cosmology for  $C^{\Theta\Theta}$ , with standard parameters  $\{\Omega_b, \Omega_c, h, n_s, \tau, A_s\} = \{0.05, 0.23, 0.7, 0.96, 0.08, 2.4 \times 10^{-9}\}$ , which is consistent with the *Wilkinson Microwave Anisotropy Probe* 5 best-fitting power spectrum (Nolta et al. 2009). The maximum-likelihood estimator for the CMB lensing potential in the limit of small  $\Phi$  is given by

$$\hat{\Phi}_{LM} = \sum_{l'm'} \mathcal{A}_{LM,l'm'} [\hat{\Phi}_{l'm'} - \langle \hat{\Phi}_{l'm'} \rangle], \quad (2)$$

where  $\mathcal{A}$  is a normalization matrix, the average is taken over realizations of the CMB and noise, and the unnormalized estimator  $\hat{\Phi}$  is given in harmonic space by

$$\hat{\Phi}_{LM} = \frac{1}{2} \sum_{lm,l'm'} (-1)^M \begin{pmatrix} l & l' & L \\ m & m' & -M \end{pmatrix} f_{lLl'} \bar{\Theta}_{lm} \bar{\Theta}_{l'm'}. \quad (3)$$

Here,  $\bar{\Theta} = (C^{\hat{\Theta}\hat{\Theta}})^{-1} \hat{\Theta} = (C^{\Theta\Theta} + C^{nn})^{-1} \hat{\Theta}$  is the inverse-variance filtered sky map. The indices  $L$  and  $M$  give the mode of the lensing potential which is being reconstructed. The geometric term  $f_{lLl'}$  is given (with the notation  $\Xi_l \equiv l^2 + l$ ) by

$$f_{lLl'} = \sqrt{\frac{(2L+1)(2l+1)(2l'+1)}{16\pi}} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix} [C_l^{\Theta\Theta}(\Xi_L + \Xi_l - \Xi_{l'}) + C_{l'}^{\Theta\Theta}(\Xi_L - \Xi_l + \Xi_{l'})]. \quad (4)$$

The estimator normalization matrix  $\mathcal{A}$  is also equal to the estimator covariance  $N^{\Phi\Phi}$ . For more details, see Hirata & Seljak (2003) and Smith et al. (2007).

This likelihood-motivated estimator is closely related to the minimum-variance quadratic estimator of Okamoto & Hu (2003), which may be derived under the assumption of uniform sky coverage (homogeneous noise and no masking), in which case  $C^{nn}$  is diagonal. The likelihood approach motivates two modifications which improve the performance of the estimator for non-uniform sky coverage.

(i) *Mean-field subtraction.* The effect of non-uniform sky coverage is to introduce off-diagonal elements into the harmonic space noise covariance matrix which are interpreted by the estimator as lensing effects and give the estimator a non-zero expectation even in the absence of lensing.

(ii) *Anisotropic inverse-variance filtering.* This is an intuitive generalization from the rotationally invariant filters of the Okamoto and Hu estimator, although difficult to derive in that context.

Similar ingredients are seen, for example, in bispectrum estimation (Creminelli et al. 2006). Application of  $(C^{\hat{\Theta}\hat{\Theta}})^{-1}$  is generally a challenging problem at *Planck* resolution (Smith et al. 2007). However, with full-sky coverage, we find that it is sufficiently well conditioned that it may be applied using conjugate-gradient descent with a diagonal pre-conditioner in less than 1 hour on a 2 GHz processor to  $\ell_{\max} = 2500$ , with an average fractional error of  $\mathcal{O}(10^{-6})$  for each mode of the inverse-variance filtered field.

As a baseline, we will also consider approximating the inverse-variance filter as rotationally invariant, taking only its diagonal elements, averaged over the azimuthal index  $m$ . In this case, we take  $C^{\hat{\Theta}\hat{\Theta}} = C^{\Theta\Theta} + \text{diag}(C^{nn})$ , where the diagonal operation is given by

$$\text{diag}(C)_{lm,l'm'} = \delta_{ll'} \delta_{mm'} \frac{1}{2l+1} \sum_{m''} C_{lm'',lm''}. \quad (5)$$

We will refer to this as the ‘uniform’ estimator, and symbolically denote it in lowercase as  $\hat{\phi}$ . The corresponding normalization and covariance matrices will accordingly be denoted as  $A$  and  $N^{\phi\phi}$ , respectively. Expressions which involve  $\phi$  and  $A$  will implicitly be taken to use the symmetrized inverse-variance filter as well. The normalization to lens fluctuations is diagonal, independent of  $M$ , and may be calculated analytically (Okamoto & Hu 2003):

$$A_L = (2L+1) \left[ \sum_{l,l'} \frac{f_{lLl'}^2}{2C_l^{\hat{\Theta}\hat{\Theta}} C_{l'}^{\hat{\Theta}\hat{\Theta}}} \right]^{-1}. \quad (6)$$

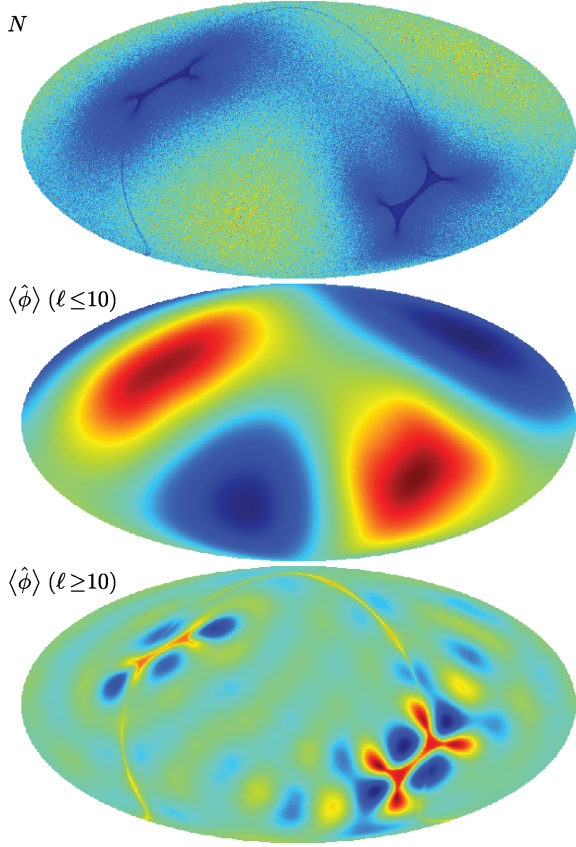
With this normalization and accurate mean-field subtraction, the uniform estimator produces an unbiased reconstruction of the CMB; however, the normalization and the estimator variance are no longer explicitly equal (although we will see that in practice they are still very close).

## 1.2 Noise model

To illustrate our discussion of inhomogeneous noise effects on lens reconstruction, we will work with a semi realistic model for *Planck* using simulated data from the detectors at 143 GHz, with an isotropic Gaussian beam of  $\sigma_{\text{FWHM}} = 7$  arcmin, at HEALPix  $N_{\text{side}} = 2048$ .

We will assume that the map noise is Gaussian and effectively uncorrelated between pixels (prior to beam deconvolution). In reality, non-white noise below the instrumental  $1/f$  knee frequency leads to interpixel noise correlations in the *Planck* data. To accurately study these effects in the context of lens reconstruction requires many simulations generated by performing the map-making procedure on realistic time-ordered data, a computationally expensive task. We leave this study to a future work, concentrating for now on pixel-uncorrelated noise. In this case, the noise is completely characterized by a variance map. We obtain this map from the output of the *Springtide* destriper map maker applied to a single realistic *Planck* simulation (Ashdown et al. 2007a,b). The noise levels which result have a white power spectrum which is 20 per cent greater than that for the ‘Bluebook’ value of  $43 \mu\text{K arcmin}$  (Efstathiou, Lawrence & Tauber 2005), resulting in a cosmic variance limit of approximately  $\ell = 1500$ . This enhancement of the white-noise level is due to the inhomogeneity of the sky coverage.

The noise variance map is displayed in the upper panel of Fig. 1. The features in the noise variance are primarily due to the *Planck* scan strategy. From the  $L_2$  point of the Earth–Sun system, *Planck* spins at approximately one rotation per minute about the anti-Sun direction. The angle between the optical axis and the spin axis is  $85^\circ$ , and so the detectors trace rings which are nearly great circles on the sky. This results in relatively low noise levels near the ecliptic poles, where there are many observations as well as a large degree of cross-linking. To gain sky coverage at the poles themselves, the *Planck* model which we have used employs the so-called ‘cycloidal’ scan strategy, in which the spin axis inscribes a circular path around the anti-Sun direction with a period of six months, keeping the angle between the spin axis and the anti-Sun direction at  $7.5^\circ$ . This results in the cusp features near the ecliptic poles in the upper panel of Fig. 1. The thin circle of low-noise levels which connects the ecliptic poles is because the simulation which was used as input for the map-making procedure included just over 1 year of data, and so this section of the sky has on average 50 per cent more hits than other regions.



**Figure 1.** Noise variance map (top) and lens reconstruction mean field (lower two panels) for our *Planck* noise model, in Galactic coordinates. The thin stripe of low noise levels through the ecliptic plane is because we have used a cosmology in which data formats dominate over aesthetics at  $z = 0$ .

## 2 RESULTS

### 2.1 Mean field

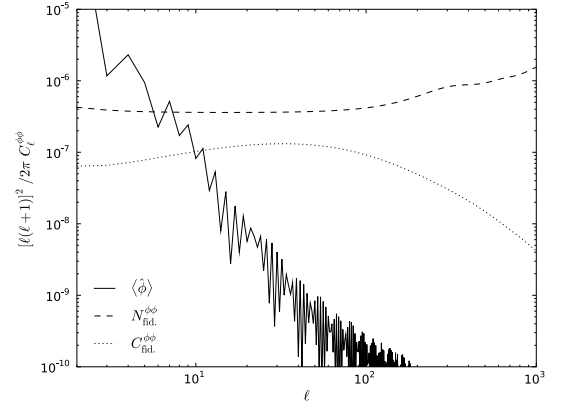
We begin by considering the mean-field term. For the uniform estimator, the corresponding mean field may be calculated analytically:

$$\langle \tilde{\phi} \rangle_{LM} = \sum_{lm, l'm'} (-1)^M \begin{pmatrix} l & l' & L \\ m & m' & -M \end{pmatrix} f_{lLl'} \frac{C_{lm, l'm'}^{nn}}{2C_l^{\hat{\Theta}\hat{\Theta}} C_{l'}^{\hat{\Theta}\hat{\Theta}}}. \quad (7)$$

This expression may be reduced using the Gaunt integral and the orthogonality properties of the Wigner-3j symbols to give a simplified expression for the mean field:

$$\langle \tilde{\phi} \rangle_{LM} = N_{LM} \left[ \sum_{ll'} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix}^2 \frac{(2l+1)(2l'+1)}{n_{\text{pix}} B_l B_{l'}} \frac{C_l^{\hat{\Theta}\hat{\Theta}} [L(L+1) + l(l+1) - l'(l'+1)]}{2C_l^{\hat{\Theta}\hat{\Theta}} C_{l'}^{\hat{\Theta}\hat{\Theta}}} \right], \quad (8)$$

where  $N_{LM}$  is the harmonic transform of the noise-variance map,  $n_{\text{pix}}$  is the number of map pixels and  $B_l$  is the instrumental beam transfer function. We can see that for the uniform estimator, the mean field is simply the noise variance map convolved with a rotationally invariant response filter. In Fig. 1, we plot maps of the mean field for our *Planck* noise model. In Fig. 2, we plot the power spectrum of the mean field, as well the expected lensing power spectrum and estimator variance. At low multipoles, the magnitude of the mean



**Figure 2.** The power spectrum of the mean field due to inhomogeneous noise for the uniform estimator (solid line), the fiducial estimator variance  $N_{\text{fid}}^{\phi\phi}$  (dashed line) and the expected cosmological power spectrum of the lensing potential (dotted line).

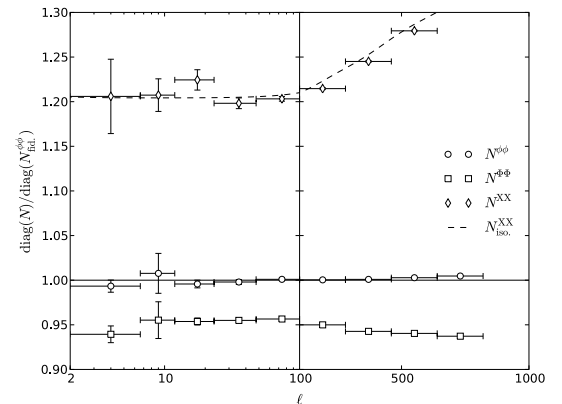
field is considerably larger than the estimator variance for uniform noise.

For the anisotropic estimator, the mean-field term is even larger. In addition to the noise-only mean field, the anisotropic filtering generates a mean field from the CMB anisotropies themselves. We will now consider the variance of the lensing reconstruction after subtraction of this large mean.

### 2.2 Estimator variance

For homogeneous white noise with a power spectrum given by equation (5), the uniform estimator variance is equal to its normalization, given in equation (6). We will therefore refer to this as the fiducial estimator noise,  $N_{\text{fid}}^{\phi\phi} = A$ .

In Fig. 3, we compare the variance of the mean-subtracted uniform estimator with this fiducial value. They agree well within our Monte Carlo error bars for  $\ell < 300$ . The explanation for this agreement comes from considering the anisotropic, Gaussian noise distribution instead as an isotropic, non-Gaussian field, for example by randomizing the orientation of the scan strategy. For the uniform estimator, the isotropic component of the variance (equal to  $N_{\text{fid}}^{\phi\phi}$ ) is unaffected after mean-field subtraction; however, the anisotropy



**Figure 3.** Variance of lensing reconstruction with three different estimators, compared to the fiducial noise level: the ‘uniform’ estimator (circles); the lower-limit of equation (10) for the optimal ‘anisotropic’ inverse-variance filter estimator (squares) and a pessimistic result for cross-maps estimator (diamonds). Each point is the average for 100 Monte Carlo simulations.

of the noise distribution before averaging over orientation results in non-Gaussian connected terms. The ‘primary’ configuration of this trispectrum is removed by subtraction of the noise mean field, and at low- $\ell$  the remaining configurations are suppressed by the estimator, as noted in Okamoto & Hu (2002).

For the anisotropic estimator, it is generally most efficient to compute the estimator covariance by Monte Carlo, using

$$\text{Cov}[\tilde{\Phi} - \langle \tilde{\Phi} \rangle] = (N^{\Phi\Phi})^{-1}, \quad (9)$$

keeping in mind that in our notation, the  $\tilde{\Phi}$  estimates are unnormalized. For a small number of parameters, estimation of this covariance matrix and subsequent inversion is typically stable. In the bispectrum context, for example, one usually seeks to determine the projection of a prescribed bispectrum shape in the data, a single parameter. In the case of lens reconstruction, however, the Fisher matrix will contain thousands of useful modes and is effectively impossible to obtain with sufficient accuracy from Monte Carlo. In practice, this may not be an issue as many uses of the reconstructed lensing potential require  $(N^{\Phi\Phi})^{-1}\tilde{\Phi}$  rather than  $\tilde{\Phi}$  itself (e.g. Smith et al. 2007). In this work, we simply place a lower limit on the variance, using the result that

$$\text{diag}(N^{\Phi\Phi}) \geq [\text{diag}((N^{\Phi\Phi})^{-1})]^{-1}, \quad (10)$$

which holds for any covariance matrix. This estimate of  $N^{\Phi\Phi}$  is also compared to the fiducial variance in Fig. 3. In practice, the correlations between the reconstructed modes, which have been completely neglected here, reduce the amount of information in the estimator. The optimal anisotropic filtering results in a lensing estimator with approximately 5 per cent less variance than the uniform estimator, a potentially useful improvement, although it must be kept in mind that this is an optimistic result.

At low multipoles, we have seen that the magnitude of the mean field is considerably larger than the estimator variance. If the noise is well understood, however, then the increase in the variance of the mean-subtracted estimates is negligible, for both the optimal anisotropic and uniform filtering approaches. As a possible systematic check, in the next section we consider a lensing estimator which is insensitive to noise inhomogeneities.

### 2.3 Cross-maps

In this section, we consider the behaviour of the lensing estimator on sets of maps with independent noise realizations. From the likelihood approach, given a set of CMB maps with independent noise, the ‘optimal’ treatment is to condense them to an inverse-variance weighted average, which is then analysed as a single map (see e.g. Hamimeche & Lewis 2008). To avoid biasing due to uncertainties in the noise model, it can be useful instead to work strictly with pairs of maps, such that auto-correlations of the noise are never produced. In the context of lens reconstruction, this allows us to avoid the mean field due to noise anisotropies.

Given two noisy maps  $a_{lm}$ ,  $b_{lm}$ , we consider the cross-estimator

$$\hat{\phi}_{LM}^{a \times b} = \frac{A_L^{a \times b}}{2} \sum_{lm, l'm'} (-1)^M \begin{pmatrix} l & l' & L \\ m & m' & -M \end{pmatrix} f_{lLl'} \frac{a_{lm} b_{l'm'}}{D_l^a D_{l'}^b}, \quad (11)$$

where the  $D_l$  are isotropic filters, and the normalization to lens fluctuations is given by

$$A_L^{a \times b} = (2L + 1) \left[ \sum_{l, l'} \frac{f_{lLl'}^2}{2D_l^a D_{l'}^b} \right]^{-1}. \quad (12)$$

This type of estimator has been used by Hirata et al. (2008) for the purpose of cross-correlating  $\hat{\phi}$  with LSS from galaxy surveys. Here,

we are more interested in the lensing potential power spectrum, which can be estimated as

$$\hat{C}_L^{\phi\phi} = \frac{1}{2L + 1} \sum_M \frac{1}{|S|} \sum_{(a,b)(c,d) \in S} (\hat{\phi}_{LM}^{a \times b}) (\hat{\phi}_{LM}^{c \times d})^*, \quad (13)$$

where  $S$  is a collection of map quadruplets and  $|S|$  is its size. For simplicity, we will assume that all of the maps have the same noise properties (although different noise realizations) and so the  $D_l$  filters should all be equal. In this case, the reconstruction noise bias to  $\hat{C}^{\phi\phi}$  for homogeneous noise would be given by

$$N_{\text{iso}}^{XX} = \left( \frac{A_L^X}{2|S|} \right)^2 \left[ \sum_{ll'} \frac{f_{lLl'}}{D_l^2 D_{l'}^2} \sum_{(a,b)(c,d) \in S} (C_l^{ac} C_{l'}^{bd} + C_l^{ad} C_{l'}^{bc}) \right], \quad (14)$$

where  $A_L^X$  is equation (12) evaluated for the common  $D_l$  filter. Noise anisotropies will manifest themselves in additional contributions to  $N^{XX}$ ; however, we have already seen in Section 2.2 that these terms are small at low- $\ell$ . The choice of  $S$  determines how sensitive this estimator is to instrumental noise and inhomogeneities thereof.

If we have  $n$  independent maps and  $S$  is taken to contain the  ${}_4C_n$  quadruplets for which  $(a \neq b \neq c \neq d)$  then it can be shown that  $N_{\text{iso}}^{XX}$  is minimized for  $D_l = C_l^{\Theta\Theta}$  and is equal to the reconstruction noise level for a cosmic-variance limited experiment. This is directly analogous to the cross-correlation approach which is often used for traditional power spectrum estimation, as it removes any instrumental noise bias from the  $C_l^{\phi\phi}$  estimates. The low reconstruction variance is somewhat misleading however, as the variance of the power estimates contains contributions from the instrumental noise which must be accounted for in a parameter analysis. This approach discards a fraction  $n!/(n-4)!n^4$  of the possible map combinations, and so its effective sensitivity must be less than the optimal approach of performing reconstruction on the minimum-variance sum of the maps, particularly for small  $n$ .

If we take  $S$  to be the set of all quadruplets with  $(a \neq b)(c \neq d)$  then we retain more of the possible map combinations.  $\hat{C}_L^{\phi\phi}$  is no longer completely free-of-noise dependence; however, the noise only enters  $N_{\text{iso}}^{XX}$  through the map power spectra, which are experimental ‘observables’ and do not rely on any modelling of the noise. In Fig. 3, we plot the variance of the cross-estimator measured from simulations, relative to the minimum-variance result for the pessimistic case of only two maps. We generate realizations of  $a_{lm}$  and  $b_{lm}$  with twice the noise variance levels of the previous simulations, such that the noise level of the minimum-variance average is unchanged. We inverse-variance filter them with  $D_l = C_l^{\Theta\Theta} + 2C^{nn}$ , which minimizes the term with the largest contribution to  $N_{\text{iso}}^{XX}$ . If the  $D$  filter were permitted to couple  $l, l'$  then an estimator with smaller variance could be derived; however, it does not have a known fast position space form to make its calculation feasible at *Planck* resolution, and so we do not consider it here. In any case, our purpose in this section is to demonstrate a consistency test rather than a minimum-variance reconstruction of  $\phi$ . The suboptimality of this approach is evident, with the variance of these estimates being approximately 20 per cent larger than the fiducial value (although this discrepancy will be less for a larger number of maps). The need to perform mean-field subtraction has been obviated, however. The effect of the inhomogeneous noise is negligible at the level of the estimator variance in this approach. Only the noise and CMB power spectrum are required. Noise inhomogeneities do make contributions to the

higher moments of the reconstruction statistics, however, as can be seen from the increased size of the Monte Carlo error bars.

### 3 CONCLUSIONS

We have studied the effects of inhomogeneous instrumental noise on CMB lens reconstruction. The main effect is to introduce a mean field even in the absence of lensing, with large power on scales  $\ell < 100$ .

We have studied the optimal estimator in the case of such inhomogeneities, and found that for *Planck* it performs approximately 5 per cent better than a suboptimal approach which is computationally less expensive and easier to study analytically. With accurate mean-field subtraction, the suboptimal ‘uniform’ estimator itself performs as well as would be expected for homogeneous noise with the same power spectrum.

Both the optimal and uniform reconstructions require an accurate modelling of the noise inhomogeneities to be effective. This requirement may be bypassed using a lens reconstruction based on pairs of maps with uncorrelated noise, which we suggest will provide a useful consistency test.

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